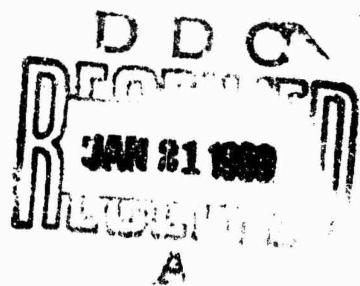


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GRAVITY ASSIST FROM JUPITER'S MOONS FOR JUPITER-ORBITING SPACE MISSIONS

R. W. Longman



PREPARED FOR:

UNITED STATES AIR FORCE PROJECT RAND

The RAND Corporation
SANTA MONICA • CALIFORNIA

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PREFACE

This Memorandum was prepared as a part of RAND's continuing research on selected problems in space trajectory mechanics and astrodynamics. In recent years many studies of interplanetary trajectories have described the use of a close approach to an intermediate planet to obtain savings in fuel or time in transfers to a target planet. A logical extension of this technique is the use of the target planet's moon(s) to save fuel upon arrival at the planet. Although the present study is concerned with the usefulness of the moons of Jupiter in effecting such savings, the formulas derived and the techniques used are sufficiently general that they could also be applied to similar investigations of the use of the moons of other planets, such as those of Saturn and Neptune.

The author is a consultant to the RAND Corporation.

SUMMARY

The energy and direction of motion of a space vehicle can be changed significantly by a close encounter with a moving gravitational body. This fact suggests that a spacecraft making a hyperbolic approach to a planet might use the gravitational attraction of a planetary moon in order to transfer to an elliptical orbit about that planet without expending fuel. This study investigates the use of the four large moons of Jupiter to effect the capture of a spacecraft by Jupiter.

In order to analyze the effectiveness of these four moons in the spacecraft's transfer to an elliptical orbit, one must determine the energy loss required by the spacecraft and compare it to the energy loss obtainable from a moon flyby. After a Hohmann transfer from earth, a space vehicle's specific energy (energy per unit mass) with respect to Jupiter is roughly $16 \text{ km}^2/\text{sec}^2$; it would be higher after a faster transfer. It is assumed that the most desirable final elliptical orbit about Jupiter is highly eccentric, since such an orbit allows close observation of the planet at pericenter without requiring a large energy loss. The eccentricity of the final orbit is limited only by the length of the orbital period. For an orbit with a period of 100 days, the energy is $-10.21 \text{ km}^2/\text{sec}^2$. If the period is extended to 360 days--perhaps the longest period that could be considered acceptable--the energy becomes $-4.35 \text{ km}^2/\text{sec}^2$. Thus an energy loss of at least $20 \text{ km}^2/\text{sec}^2$, and possibly much more, is necessary for a transfer from a high-thrust approach trajectory to an acceptable final orbit. The maximum possible energy loss obtainable from a moon flyby, for an incoming energy of approximately $16 \text{ km}^2/\text{sec}^2$, is slightly above $9 \text{ km}^2/\text{sec}^2$ (for the moon Ganymede) and is significantly less if the approach trajectory is not optimal. Thus, even with a moon swingby some chemical retrothrust is necessary to obtain the desired transfer. The velocity changes which result from a flyby are on the order of 0.54 km/sec at the moon's distance from Jupiter. The importance of such a free velocity change depends on the particular rocket used. Of course, a smaller velocity change produced chemically

could result in the same energy loss if it were applied closer to Jupiter.

Gravity assist from a moon becomes more promising when low-thrust trajectories are considered. Optimum low-thrust trajectories, designed to make the spacecraft pass close to Jupiter, typically have approach energies lower than the Hohmann value given above (for roughly comparable flight times), and this energy is further reduced if there is some retrothrusting during the later portions of the transfer. If the incoming energy is reduced to about $6 \text{ km}^2/\text{sec}^2$, a swingby can give an energy change slightly above $10 \text{ km}^2/\text{sec}^2$. In this case a no-impulse capture could result in a final orbit with a one-year orbital period. If such a trajectory is compared with a low-thrust trajectory designed to place the spacecraft directly into an elliptical orbit, the moon's contribution to the necessary energy change is found to relax the transfer endpoint conditions and allow shorter flight times or increased payloads for any given low-thrust vehicle.

The above figures are given for optimum encounters. The energy changes obtainable from moon flybys are degraded by variations in the distance of closest approach to the moon, in the approach angle, and in the plane of the approach orbit. The sensitivity of the energy change to these factors is considered here. Also, timing and aiming requirements are indicated for an approach energy of $5 \text{ km}^2/\text{sec}^2$.

Since the energy changes obtainable in one-moon encounters are not as large as might be needed, several two-moon encounters are considered. The much more stringent timing and aiming requirements for these are not treated here.

It might be added that trajectories using a moon flyby have the advantage of allowing observation of the moon as well as of Jupiter. Since it is sometimes suggested that these moons might be used as landing sites, observation of them might well be of interest.

ACKNOWLEDGMENTS

The author wishes to thank Dr. A. M. Schneider of the University of California, San Diego, for bringing the problem considered here to the author's attention and for helpful discussions and suggestions. The assistance of J. R. Cowley, Jr. in deriving Eq. (5) is also appreciated.

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SYMBOLS

A = the angle between the spacecraft's velocity vector with respect to Jupiter as it approaches the moon and the moon's velocity vector with respect to Jupiter (see Fig. 1).

A' = the angle between the spacecraft's velocity vector with respect to Jupiter as it departs from the moon and the moon's velocity with respect to Jupiter (see Fig. 1).

A^* = the angle between \bar{v}_{JS}^* and \bar{v}_{JM}^* ($0 \text{ deg} \leq A^* \leq 180 \text{ deg}$).

$A^{*'}$ = the angle between $\bar{v}_{JS}^{*'}$ and $\bar{v}_{JM}^{*'}$ ($0 \text{ deg} \leq A^{*' \leq 180 \text{ deg}}$).

A_{opt} = the angle of approach A which produces the largest energy decrease for a constant initial energy.

a = semimajor axis of an orbit. It is taken as positive for elliptic and negative for hyperbolic orbits.

E = the initial energy of the spacecraft with respect to Jupiter.

ΔE , $\Delta |\bar{v}|$ = changes in the energy and the magnitude of the velocity vector resulting from a moon encounter.

$M = r_c / r_m$ = the ratio of the distance of closest approach to the moon and the moon's radius, i.e., the miss ratio.

Q = the angle between the moon's velocity vector with respect to Jupiter and the spacecraft's asymptotic direction of approach to the moon (see Fig. 1).

q = Jupiter central angle from pericenter of the spacecraft orbit to the moon's position.

q_N = nominal value of q .

r_{JM} = the moon's orbital radius.

v_c , v_p = the circular and parabolic speeds at the surface of the moon.

\bar{v}_∞^1 , \bar{v}_∞^2 = the spacecraft velocity vector with respect to the moon as it approaches and departs respectively from the sphere of influence of the moon.

v_∞ = the magnitude of \bar{v}_∞^1 or \bar{v}_∞^2 .

\bar{v}_{JM} = the velocity of the moon with respect to Jupiter.

\bar{v}_{JS} , \bar{v}'_{JS} = the velocity of the satellite with respect to Jupiter as it approaches the moon and departs from the moon respectively.

\bar{v}^*_{JS} , $\bar{v}^{*'_{JS}}$, \bar{v}^*_{JM} = the vectors obtained by projecting \bar{v}_{JS} , \bar{v}'_{JS} , and \bar{v}_{JM} onto the spacecraft's orbital plane about the moon.

v^*_{JS} , $v^{*'_{JS}}$, v^*_{JM} = the magnitudes of \bar{v}^*_{JS} , $\bar{v}^{*'_{JS}}$, and \bar{v}^*_{JM} .

α = the ratio of the spacecraft's velocity at the moon's orbital distance from Jupiter to the circular velocity at that distance.

β = the angle between the moon's orbital plane and the plane of the satellite's hyperbola about the moon.

Δ = the perpendicular distance from the moon to the spacecraft hyperbolic approach asymptote to the moon (see Fig. 2).

e = eccentricity.

η = the angle between \bar{v}_{JM} and the plane of the spacecraft's hyperbolic orbit about the moon (see Fig. 4).

μ_m , μ_j = the gravitational constants for the moon and Jupiter respectively.

v , v_{max} = the angle between $\bar{v}_{\infty 1}$ and $\bar{v}_{\infty 2}$ (see Fig. 2). v_{max} is the maximum value v can have without the spacecraft colliding with the moon's surface.

φ , φ_{min} = the half angle between the asymptotes of the hyperbola about the moon (see Fig. 2). φ_{min} is the smallest value of φ for which the spacecraft does not collide with the moon.

ψ = the angle between the projection of \bar{v}_{JM} onto the plane of the spacecraft's hyperbolic orbit about the moon and the semimajor axis of the hyperbola (see Figs. 3 and 4).

ψ_N = the nominal value of the angle ψ .

ω_{JM} = the angular velocity of the moon in its orbit.

I. INTRODUCTION

Many gravity-assisted trajectories have been suggested which use the close approach of an intermediate planet to perturb a trajectory going between the earth and a target planet. Considerable savings in fuel and/or time can result from using such a trajectory.⁽¹⁻⁴⁾ In fact, a proposed gravity-assisted mission passes by Jupiter, Saturn, Uranus, and Neptune and requires only 8.9 years to complete, while a direct flight to Neptune requires 30 years.⁽³⁾ A logical extension of this technique is to use a flyby of a planet's moon(s) to effect capture of the vehicle by the planet. The earth, Jupiter, Saturn, and Neptune all have sizable moons.

Two missions for the not too distant future might use moon flybys:

1. Because Jupiter is the nearest of the large planets, flights whose objective is to go into orbit around the planet would certainly be of great interest. Jupiter is extremely massive--in fact, more massive than all the other planets combined--which indicates that the fuel required to enter a low-altitude orbit is considerable. Among its many moons there are four massive ones with diameters comparable to that of Mercury and masses up to one-half that of Mercury. A flyby of one of these moons might not only decrease the minimum propulsion requirements significantly but would also afford observation of the moon itself.

2. Round-trip flights from the earth to Mars might fly by the earth's moon on the return leg of the journey in order to minimize the overall fuel requirements. Also, swingbys of the earth's moon to increase a space vehicle's energy might be used on many outward-bound interplanetary trajectories. However, the large sensitivity of the resulting trajectories to initial launch errors must be weighed against the fuel savings. Some discussion of the use of the earth's moon can be found in Ref. 4.

The purpose of this study is to investigate the usefulness of the moons of Jupiter in effecting capture of a spacecraft. A simplified model is used to approximate the maximum possible energy change as a

Table 1
ASTRONOMICAL DATA

Astro-nomical Body	Radius ^a (km)	Mass (earth = 1)	Mean Distance (planetary radii)	Orbital Period (earth solar days)	Inclination to Planetary Equator (deg)	Parabolic Speed at Surface (km/sec)	Eccen-tricity ^b
Io	1,660	0.0121	5.9	1.77	0	2.28	0.0
Europa	1,440	0.0079	9.4	3.55	0	1.99	0.0003
Ganymede	2,470	0.0259	15.0	7.15	0.2	2.83	0.0015
Callisto	2,340	0.0160	26.4	16.69	0	2.23	0.0075
Moon	1,738	0.0122	60.3	27.3	18.3	2.38	0.05490

^a For comparison purposes note that the radius of Mercury is 2,420 and its mass 0.054.

^b Values are from Ref. 6.

function of the approach energy for each of the four large moons. If the energy change is sufficient to decrease the space vehicle's energy below the zero parabolic energy level, capture is accomplished. The sensitivity of the energy change to variations in the angle of approach and the distance of closest approach to the moon is also investigated, but little attention is given to the trajectories required to obtain the energy change. None of these results in a final circular orbit, but because of the large energy decrease required to orbit at low altitudes, the most desirable final orbit is considered to be highly elliptic to afford close observation of the planet at pericenter without requiring too large an energy loss. Perturbations of the final orbit due to the presence of other moons have been ignored, as have any constraints due to possible radiation belts.⁽⁵⁾ Perturbations due to other moons can, of course, be minimized by introducing some velocity component out of the moon's orbital plane during the swingby.

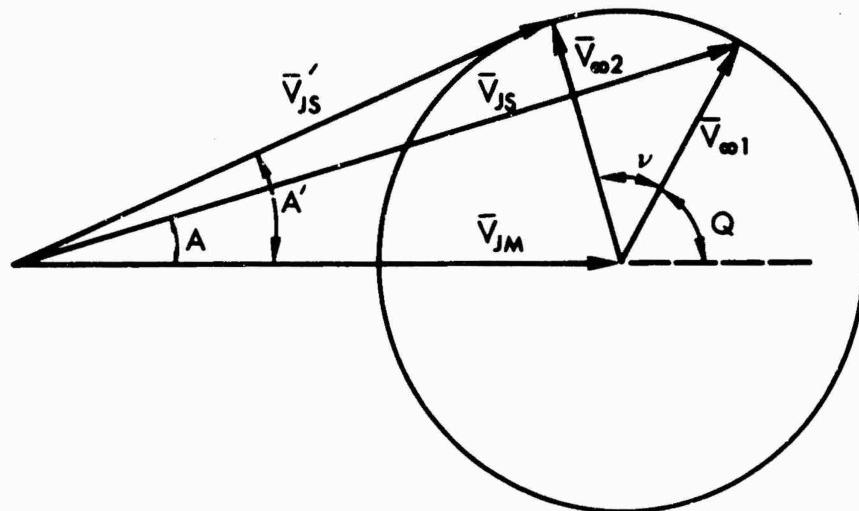
The moons' orbits are considered to be circular and coplanar, and all gravitational forces are assumed to obey a perfect inverse square law. The trajectories are treated as a series of two-body problems using the sphere-of-influence argument. In Table 1, the values used for the surface parabolic speed of each moon and its mean distance to Jupiter in planetary radii come from Ref. 7. The value used for the radius of Jupiter is 70,000 km; its mass is taken to be 318 times that of the earth.

II. ANALYTICAL BACKGROUND

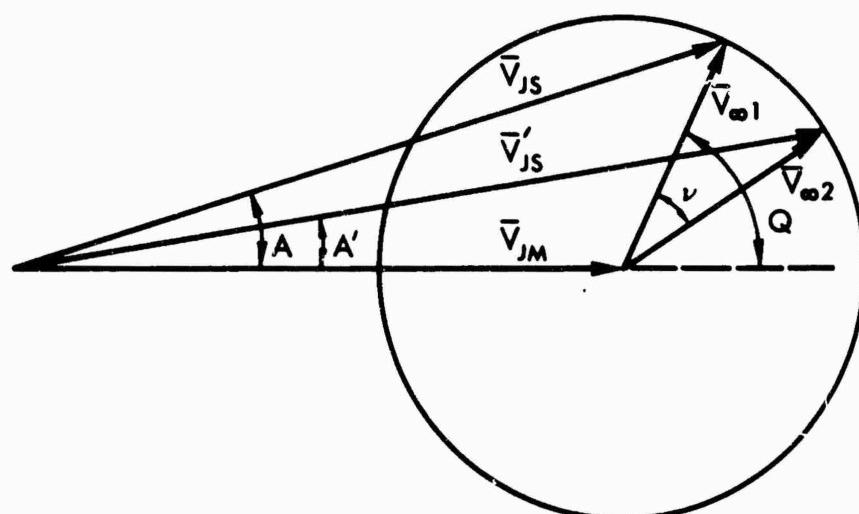
If we consider the mass of the spacecraft as negligible compared with the mass of the moon, then the motion of the moon will be unaltered by a flyby. The total potential energy field, which is the sum of the potential energies due to Jupiter and the moon, is then explicitly time-dependent because of the dependence of the moon's position on time. As a result, the total energy of the spacecraft is not conserved. Actually, the energy of the moon and spacecraft combination is conserved and energy is exchanged between the two during a flyby.

To investigate the change in energy of the vehicle, the vector diagrams shown in Fig. 1 are useful (see Ref. 8 for a similar approach). We examine Fig. 1a in detail. The vector \bar{V}_{JS} is the velocity the space vehicle has with respect to Jupiter at the moon's orbital radius from Jupiter if the gravitational attraction of the moon is not considered. We subtract the vector velocity of the moon with respect to Jupiter, \bar{V}_{JM} , to obtain the vector $\bar{V}_{\infty 1}$. The sphere of influence of the moon is considered to be negligibly small compared with the moon's orbital radius, yet large enough that it can be considered at infinity with respect to the moon. (Using the formula from Ref. 9, with an obvious correction, the sphere of influence of Ganymede is 0.347 Jupiter radii compared with an orbital radius of 15 Jupiter radii.) Under these assumptions \bar{V}_{JS} is the velocity of the satellite with respect to Jupiter as it enters the moon's sphere of influence, and the direction of $\bar{V}_{\infty 1}$ is the direction of the hyperbolic approach asymptote to the moon while its magnitude is the velocity of approach at infinity with respect to the moon. The effect of the hyperbolic encounter is to rotate the vector $\bar{V}_{\infty 1}$ through v degrees from the inbound asymptote to the outbound asymptote, and its magnitude remains unchanged. After performing this rotation and calling the new vector $\bar{V}_{\infty 2}$, we obtain the new velocity with respect to Jupiter as the spacecraft leaves the moon's sphere of influence from $\bar{V}'_{JS} = \bar{V}_{JM} + \bar{V}_{\infty 2}$. This defines the new orbit of the space vehicle and determines its new energy and angular momentum.

The magnitude of the angle v is limited by the finite size of the moon. Let us define a miss ratio $M = r_c/r_m$ where r_c is the radial



(a) Energy loss



(b) Energy gain

Fig. 1 — Vector diagram of the velocities before
and after a moon flyby

distance from the center of the moon to the point of closest approach and r_m is the radius of the moon. Clearly the choice of M is limited to $M > 1$. Using the conservation of angular momentum and energy, we obtain the perpendicular distance from the moon to an asymptote, Δ ,

$$\Delta = r_c \left[1 + \frac{1}{M} \left(\frac{V_p}{V_\infty} \right)^2 \right]^{1/2}$$

where V_p is the parabolic velocity at the surface of the moon and V_∞ is the magnitude of $\bar{V}_{\infty 1}$ and $\bar{V}_{\infty 2}$. When $M = 1$, $r_c = r_m$ and Δ becomes the collision length (see Ref. 6). From Fig. 2 we see that $\sin \varphi = \Delta / (r_c - a)$ where "a" is the semimajor axis of the hyperbola, which is taken as a negative quantity. Under the assumptions stated above, the energy of the vehicle with respect to the moon is given as

$$E = \frac{1}{2} V_\infty^2 = - \frac{\mu_m}{2a}$$

where μ_m is the product of the universal constant of gravitation and the mass of the moon. Thus we can write

$$\varphi = \sin^{-1} \left[\frac{\Delta}{r_c + \frac{\mu_m}{V_\infty^2}} \right]$$

which can be written as

$$\varphi = \cos^{-1} \left[\frac{1}{1 + 2M \left(\frac{V_\infty}{V_p} \right)^2} \right] \quad (1)$$

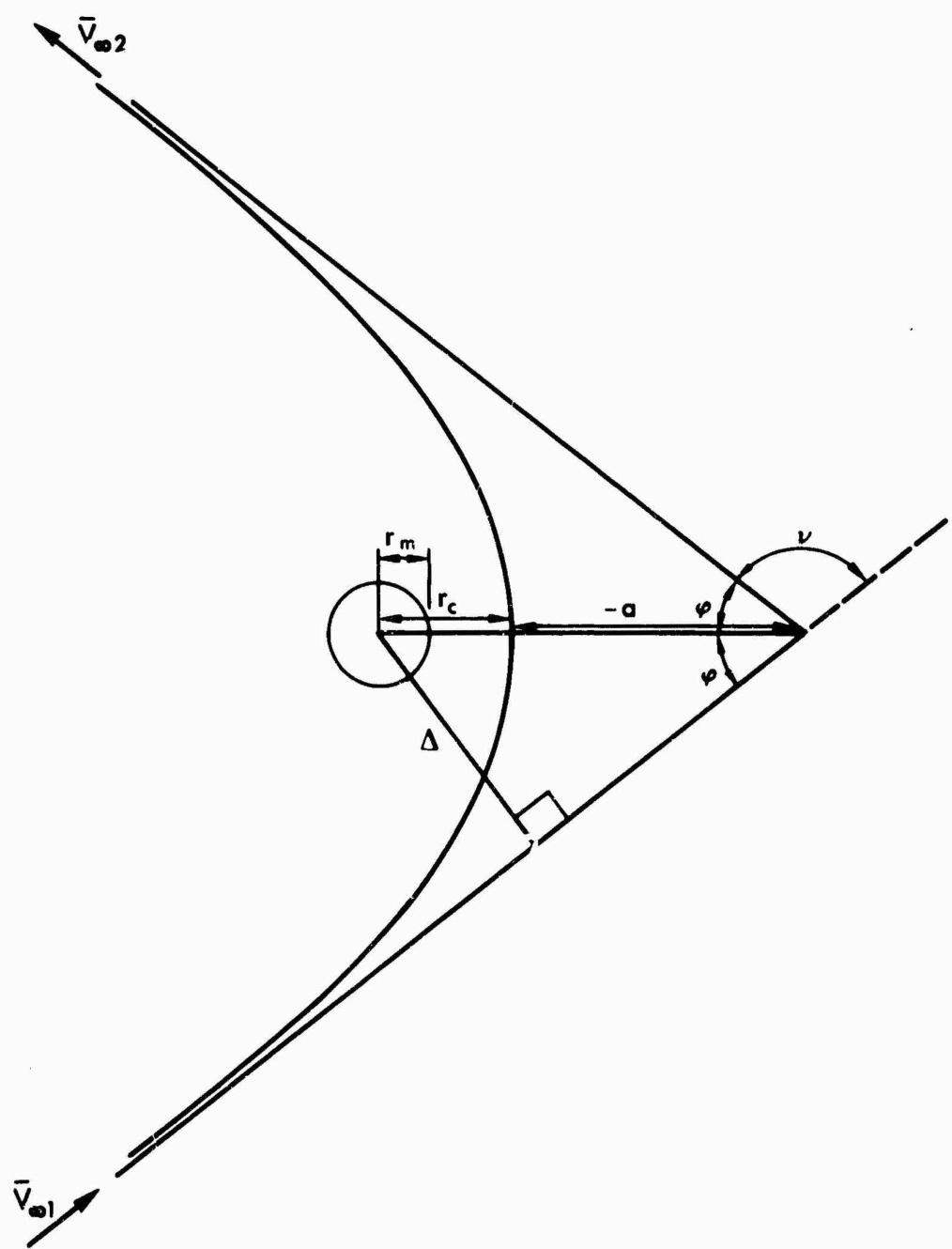


Fig. 2—Hyperbolic encounter with a moon

Knowing φ , we obtain $v = 180 \text{ deg} - 2\varphi$.

By adding v to the angle Q as in Fig. 1a we obtain a decrease in energy, which corresponds to a hyperbolic trajectory that crosses the moon's orbit ahead of the moon. If v is subtracted from Q as in Fig. 1b, the energy of the vehicle increases. In this case the spacecraft crosses the moon's orbit behind the moon. In general, in order to maximize the energy change, we want v to be as large as possible (v_{\max}), or equivalently φ to be as small as possible (φ_{\min}). This is accomplished by letting M approach 1 so that the spacecraft comes within an infinitesimal distance of the moon's surface. There are, however, some cases in which the maximum energy change is obtained for a $v \neq v_{\max}$. Figure 1a shows that if $v_{\max} + Q > 180 \text{ deg}$, the final velocity is minimized and the energy change maximized when $v = 180 \text{ deg} - Q$. Any value of v that is less than v_{\max} can be obtained by increasing the miss ratio M , since this gives all values of φ , $\varphi_{\min} < \varphi < 90 \text{ deg}$. When the best value of v is given by $v = 180 \text{ deg} - Q$, we see that \bar{V}'_{JS} , \bar{V}_{JM} , and $\bar{V}_{\infty 2}$ are all colinear and that the magnitude of the velocity after an encounter is given by $V'_{JS} = |V_{JM} - V_{\infty}|$. Its direction is parallel to \bar{V}_{JM} if $V_{JM} > V_{\infty 2}$ and opposite to \bar{V}_{JM} if $V_{JM} < V_{\infty 2}$. Similarly for the case of energy addition if $v_{\max} > Q$, the best value of v is not v_{\max} but $v = Q$, which gives $V'_{JS} = V_{JM} + V_{\infty}$. These special cases occur for energy subtraction when the approach velocity V_{JS} is smaller than the moon's velocity and the approach angle is small, and for energy addition when the approach velocity is larger than the moon's velocity and the approach angle is small.

Since spacecraft performance is often considered in terms of a "ΔV budget," it is of interest to examine the maximum possible change in the magnitude of the velocity vector $|\Delta\bar{V}|$ which can occur in a moon flyby. To do this we first consider the maximum magnitude of the vector change in the velocity $|\Delta\bar{V}|$ which we recognize as an upper bound for $|\Delta\bar{V}|$. From trigonometric considerations

$$|\Delta\bar{V}| = 2V_{\infty} \cos \varphi$$

Maximizing with respect to V_∞ after using Eq. (1) gives

$$\max_{[V_\infty]} |\Delta\bar{v}| = \frac{v_c}{\sqrt{M}}$$

where v_c is the circular velocity at the surface of the moon. The maximum occurs when

$$V_\infty = \frac{v_c}{\sqrt{M}} \quad (2)$$

and Eq. (1) indicates that $\varphi = v = 60$ deg independent of M . Examining Figs. 1a and 1b, we see that unless $v_c/\sqrt{M} > V_{JM}$ there will exist a vector \bar{v}_{JS} such that the angles A and A' are equal and \bar{v}_{JS} becomes colinear with \bar{v}'_{JS} . In this case all of the $|\Delta\bar{v}|$ can be realized as $\Delta|\bar{v}|$, so that the maximum possible change in the magnitude of the velocity vector is

$$\max_{[V_\infty]} \Delta|\bar{v}| = \frac{v_c}{\sqrt{M}} \quad (3)$$

If M is allowed to approach 1, the result is that the maximum velocity change obtainable from a flyby is equal to the circular velocity at the surface of the moon. (For Ganymede, the largest moon of Jupiter, the circular velocity is 2.0 km/sec.) However, this condition occurs at an approach energy well below the range of interest, which is roughly between 0 and $20 \text{ km}^2/\text{sec}^2$, and more realistic values for the $\Delta|\bar{v}|$ lie between 0.5 and 0.7 km/sec. The approach energy which gives the maximum $\Delta|\bar{v}|$ can be determined from the approach velocity V_{JS} . By simple trigonometry the maximum decrease of $\Delta|\bar{v}|$ occurs when

$$V_{JS} = \frac{v_c}{2\sqrt{M}} + \frac{1}{2} \sqrt{4V_{JM}^2 - 3 \frac{v_c^2}{M}}$$

Converting to the case of a maximum increase by interchanging $\bar{v}_{\infty 1}$ and $\bar{v}_{\infty 2}$, \bar{v}_{JS} and \bar{v}'_{JS} , we get

$$v_{JS} = - \frac{v_c}{2\sqrt{M}} + \frac{1}{2} \sqrt{4v_{JM}^2 - 3 \frac{v_c^2}{M}}$$

which differs from the maximum decrease case by v_c/\sqrt{M} .

For purposes of the present study the change of energy is of greater interest than that of velocity. The energy change can be written as

$$\Delta E = (\Delta |\bar{v}|) v_{JS} + \frac{1}{2} (\Delta |\bar{v}|)^2$$

where $\Delta |\bar{v}|$ is taken as positive for a velocity increase and negative for a decrease. Because of the presence of v_{JS} , the approach energy at which $\Delta |\bar{v}|$ is extremized is not the same as that for ΔE . To find $\max \Delta E$ we use Fig. 3, which defines an x-, y-coordinate system in the plane of motion of the space vehicle with the x-axis fixed along the semimajor axis of the hyperbola. Temporarily we consider the velocity of the moon with respect to Jupiter \bar{v}_{JM} to lie in the xy-plane at an angle ψ from the x-axis.

Denoting a unit vector along the positive x-direction as \hat{i} and along the positive y-direction as \hat{j} , we can write

$$\bar{v}_{JM} = v_{JM} \cos \psi \hat{i} + v_{JM} \sin \psi \hat{j}$$

$$\bar{v}_{\infty 1} = v_{\infty} \cos \varphi \hat{i} + v_{\infty} \sin \varphi \hat{j}$$

$$\bar{v}_{\infty 2} = - v_{\infty} \cos \varphi \hat{i} + v_{\infty} \sin \varphi \hat{j}$$

$$\bar{v}_{JS} = \bar{v}_{JM} + \bar{v}_{\infty 1}$$

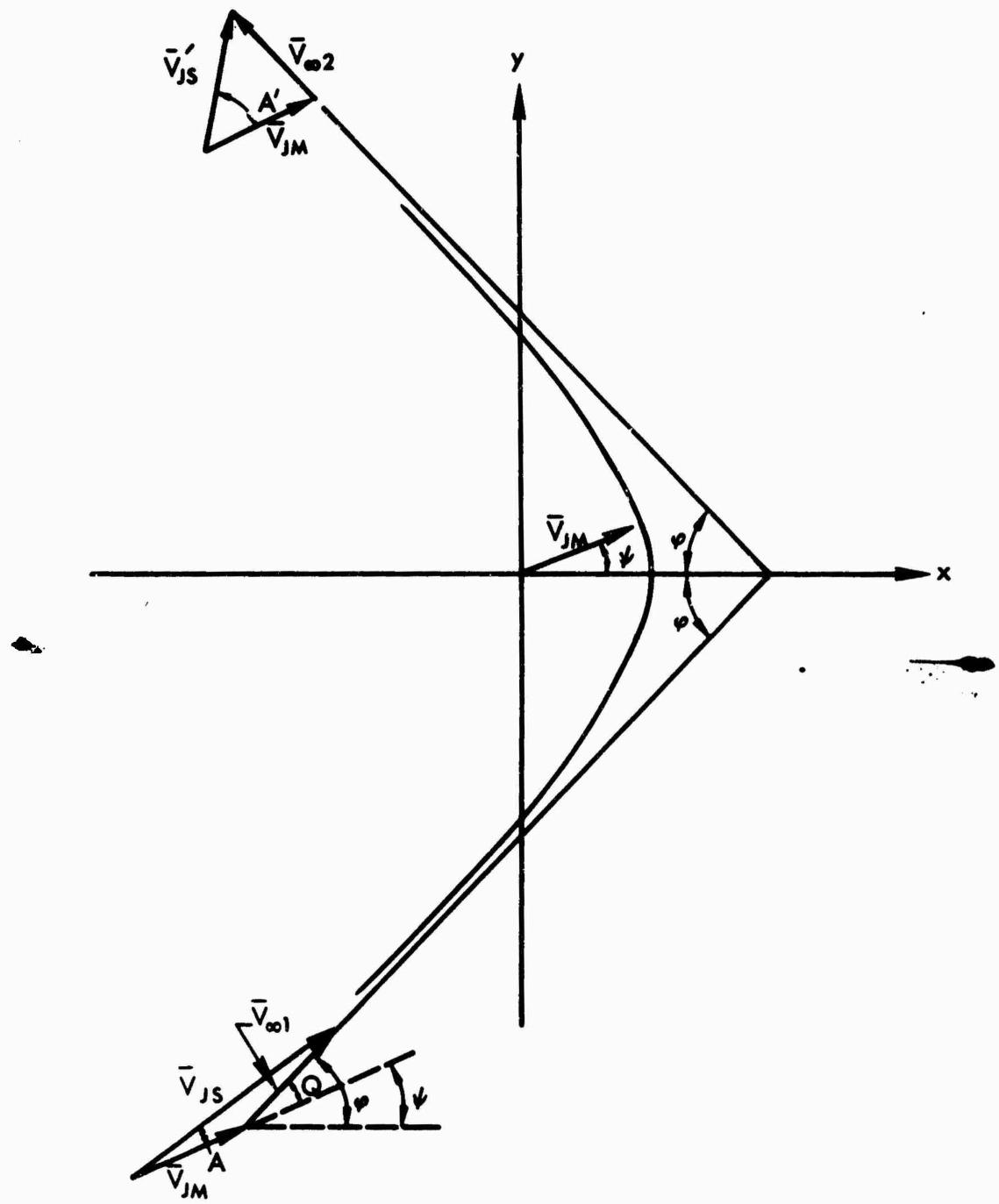


Fig. 3—Moon encounter in a coordinate system fixed to the major axis of the hyperbola. (Coplanar case)

$$\bar{v}'_{JS} = \bar{v}_{JM} + \bar{v}_{\infty 2}$$

and

$$\Delta E = \frac{1}{2} [(v'_{JS})^2 - (v_{JS})^2]$$

which gives

$$\Delta E = -2V_{JM}V_{\infty} \cos \psi \cos \varphi \quad (4)$$

We consider ΔE as a function of ψ and V_{∞} and use Eq. (1) to obtain the maximum possible energy change (1)

$$\max_{[\psi, V_{\infty}]} \Delta E = \pm V_{JM}V_c/\sqrt{M} \quad (5)$$

which occurs at $V_{\infty} = V_c/\sqrt{M}$ and $\varphi = 60$ deg as in the case of maximum $\Delta|\bar{v}|$. The sign depends on the choice of the angle ψ as 0 deg for energy decrease and 180 deg for energy addition. (Note that if ΔE is maximized for a constant energy of approach to Jupiter, these values of ψ will no longer hold.) Since $\psi = 0$ deg for energy subtraction, $Q = \varphi = 60$ deg. For energy addition, with $\psi = 180$ deg, $Q = \psi - \varphi_{\min} = 120$ deg. Note that \bar{v}_{JS} and \bar{v}'_{JS} are not colinear. Note also that the maximum energy change can never occur for a value of v which is not equal to v_{\max} . The maximum energy change occurs when

$$v_{JS} = \sqrt{\frac{1}{M} v_c^2 + v_{JM}^2 \pm \frac{1}{\sqrt{M}} V_{JM} V_c}$$

$$E = \frac{1}{2} \left(\frac{1}{M} v_c^2 + v_{JM}^2 \pm \frac{1}{\sqrt{M}} V_{JM} V_c \right) - \frac{\mu_J}{r_{JM}} \quad (6)$$

$$A = \cos^{-1} \left[\frac{v_{JS}^2 + v_{JM}^2 - \frac{1}{M} v_c^2}{2V_{JS}V_{JM}} \right]$$

where the plus sign is used in the case of energy subtraction and the minus sign for energy addition.

We now consider the problem of finding the energy change that results for any given initial energy E (or equivalently for any initial V_{JS}) and approach angle A , using Eq. (4). For energy subtraction $\Psi = \varphi - Q$, and for energy addition $\Psi = \varphi + Q$. When the best value of v is not v_{\max} , then for energy subtraction $2\varphi_{\min} < Q$ and for energy addition $v_{\max} > Q$. We write

$$\sin Q = \frac{V_{JS}}{V_{\infty}} \sin A$$

$$\cos Q = \frac{V_{JS} \cos A - V_{JM}}{V_{\infty}}$$

$$\sin \varphi = 2 \sqrt{M} \left(\frac{V_{\infty}}{V_p} \right) \left[1 + M \left(\frac{V_{\infty}}{V_p} \right)^2 \right]^{1/2} \cos \varphi$$

Combining these results we obtain

$$\Delta E = \begin{cases} \frac{-2V_{JM}}{\left[1 + 2M \left(\frac{V_{\infty}}{V_p} \right)^2 \right]^2} \left\{ V_{JS} \cos A - V_{JM} \pm 2V_{JS} \sin A \left[M \left(\frac{V_{\infty}}{V_p} \right)^2 + M^2 \left(\frac{V_{\infty}}{V_p} \right)^4 \right]^{1/2} \right\} \\ \text{when } \frac{2}{\left[1 + 2M \left(\frac{V_{\infty}}{V_p} \right)^2 \right]^2} - 1 < \pm \frac{V_{JS} \cos A - V_{JM}}{V_{\infty}} \\ \frac{1}{2} \left(V_{JM} \mp V_{\infty} \right)^2 - \frac{1}{2} V_{JS}^2 \\ \text{when } \frac{2}{\left[1 + 2M \left(\frac{V_{\infty}}{V_p} \right)^2 \right]^2} - 1 > \pm \frac{V_{JS} \cos A - V_{JM}}{V_{\infty}} \end{cases} \quad (7)$$

with

$$v_{\infty} = \sqrt{v_{JS}^2 + v_{JM}^2 - 2v_{JS}v_{JM} \cos A}$$

Whenever there is a sign choice, the upper sign is used for energy subtraction and the lower for energy addition. When ΔE is known, $\Delta|\vec{v}|$ can easily be found as

$$\Delta|\vec{v}| = \sqrt{2\Delta E + v_{JS}^2 - v_{JS}}$$

The departure angle A' is given by

$$A' = \cos^{-1} \left(\frac{\frac{v_{JM}^2 + v_{JS}^2 + 2\Delta E - v_{\infty}^2}{2v_{JM}}}{\sqrt{v_{JS}^2 + 2\Delta E}} \right)$$

Knowledge of ΔE , A' , and the position of the moon at the time of the encounter completely determines the resulting orbit around Jupiter (actually, one needs to know not only E and the magnitude of A , which determines ΔE and A' , but also whether the satellite approaches the moon from outside or inside the moon's orbital radius). It is interesting to note that the amount of energy addition possible in a swing-by starting from a given initial trajectory is in general different from the amount of energy subtraction possible starting from the same trajectory, a fact which is obvious from Fig. 1.

Equation (7) assumes that all spacecraft motion takes place in the moon's orbital plane. It is easy to generalize the equation to the case where the moon's velocity vector \vec{v}_{JM} makes an angle η with the satellite's plane of motion within the moon's sphere of influence. The component of \vec{v}_{JM} out of the xy-plane in Fig. 3 is then $v_{JM} \sin \eta$, which equals the out-of-plane components of both \vec{v}_{JS} and \vec{v}'_{JS} . Thus the out-of-plane component is unaffected by the encounter with the

moon, which implies that Eq. (7) will still hold if \bar{V}_{JM} , \bar{V}_{JS} , and A are replaced by \bar{V}_{JM}^* , \bar{V}_{JS}^* , and A^* , the values given by a projection of \bar{V}_{JM} and \bar{V}_{JS} onto the xy -plane. Figure 4 shows the geometry involved.

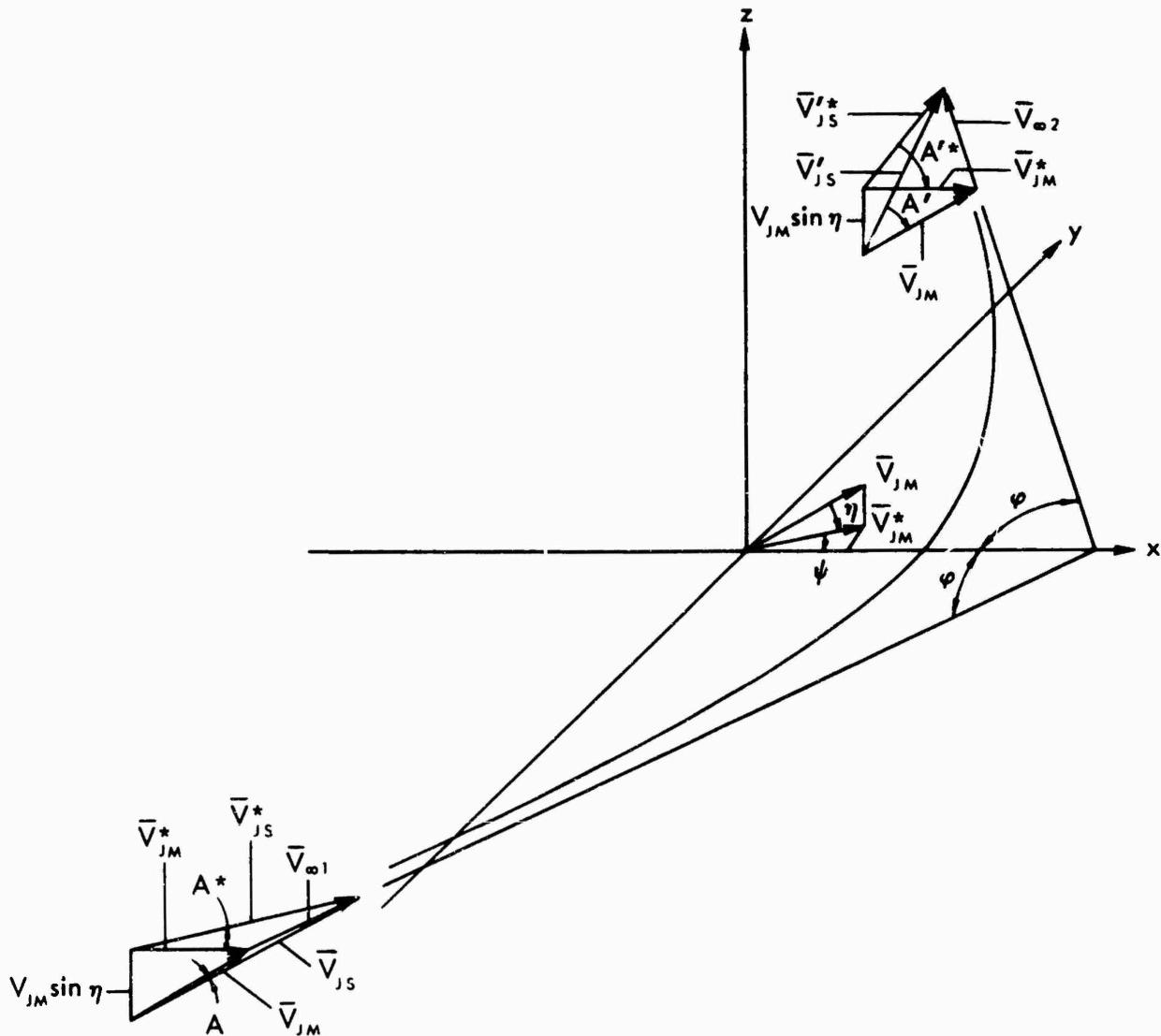


Fig. 4—Modifications of Fig. 3 when the moon's velocity vector makes an angle η with the satellite's plane of motion near the moon

The projected values are determined from

$$v_{JM}^* = v_{JM} \cos \eta$$

$$v_{JS}^* = \sqrt{v_{JS}^2 - v_{JM}^2 \sin^2 \eta}$$

$$A^* = \cos^{-1} \left[\frac{v_{JS} \cos A - v_{JM} \sin^2 \eta}{\cos \eta \sqrt{v_{JS}^2 - v_{JM}^2 \sin^2 \eta}} \right] \quad \text{with } (0^\circ \leq A^* \leq 180^\circ) \quad (8)$$

III. NUMERICAL RESULTS

ENERGY CHANGE RELATED TO INITIAL ENERGY, MISS RATIO, AND APPROACH ANGLE

Since no analytical expression was found to determine the maximum energy decrease obtainable for a given initial energy, a computer program was written which uses a numerical search technique to determine the maximum energy change and the corresponding optimum angle of approach A_{opt} . Figure 5 gives $\Delta E_{[A]}$ versus E for the four large moons. (Note this uses $M \geq 1$, $\eta = 0$. Note also that Eq. (7) and Fig. 1 indicate that whenever M is constrained to be greater than or equal to some constant, the $\Delta E_{[A]}$ will occur when M equals that constant. Thus each point on Fig. 5 occurs for $M = 1$.) The maximum point of each of the curves is given by Eq. (5), and the energy at which it occurs is given by Eq. (6). The point at which each curve gives zero energy change corresponds to the potential energy at the moon's orbital radius; it is the minimum energy a space vehicle can have and be at the moon's orbital distance from Jupiter.

The space vehicle's energy with respect to Jupiter after a Hohmann transfer from earth (assuming circular coplanar orbits) is approximately $16 \text{ km}^2/\text{sec}^2$. For a faster transfer the energy would be higher, and if a low-thrust trajectory is used, the energy could be much lower. Additional information concerning low-thrust trajectories to Jupiter is given in Ref. 10. Thus the energy range of greatest interest is roughly between $E = 0$ and $20 \text{ km}^2/\text{sec}^2$. The most desirable final orbit is perhaps a highly elliptical orbit with a reasonably short period. If the period of the orbit is taken as 60 days, the required final energy is $-14.35 \text{ km}^2/\text{sec}^2$ with a semimajor axis of 63.14 Jupiter radii. A period of 100 days gives an energy of $-10.21 \text{ km}^2/\text{sec}^2$ and semimajor axis of 88.76 Jupiter radii; if we relax the requirement to a period of 360 days, the final orbit has an energy of $-4.35 \text{ km}^2/\text{sec}^2$ and a semimajor axis of 208.48 Jupiter radii. Thus an energy loss of at least $20 \text{ km}^2/\text{sec}^2$, and possibly much more, is necessary for a higher thrust approach trajectory.

For the energy range of interest, Fig. 5 indicates that Ganymede gives the largest energy change. Obviously on a high-thrust trajectory

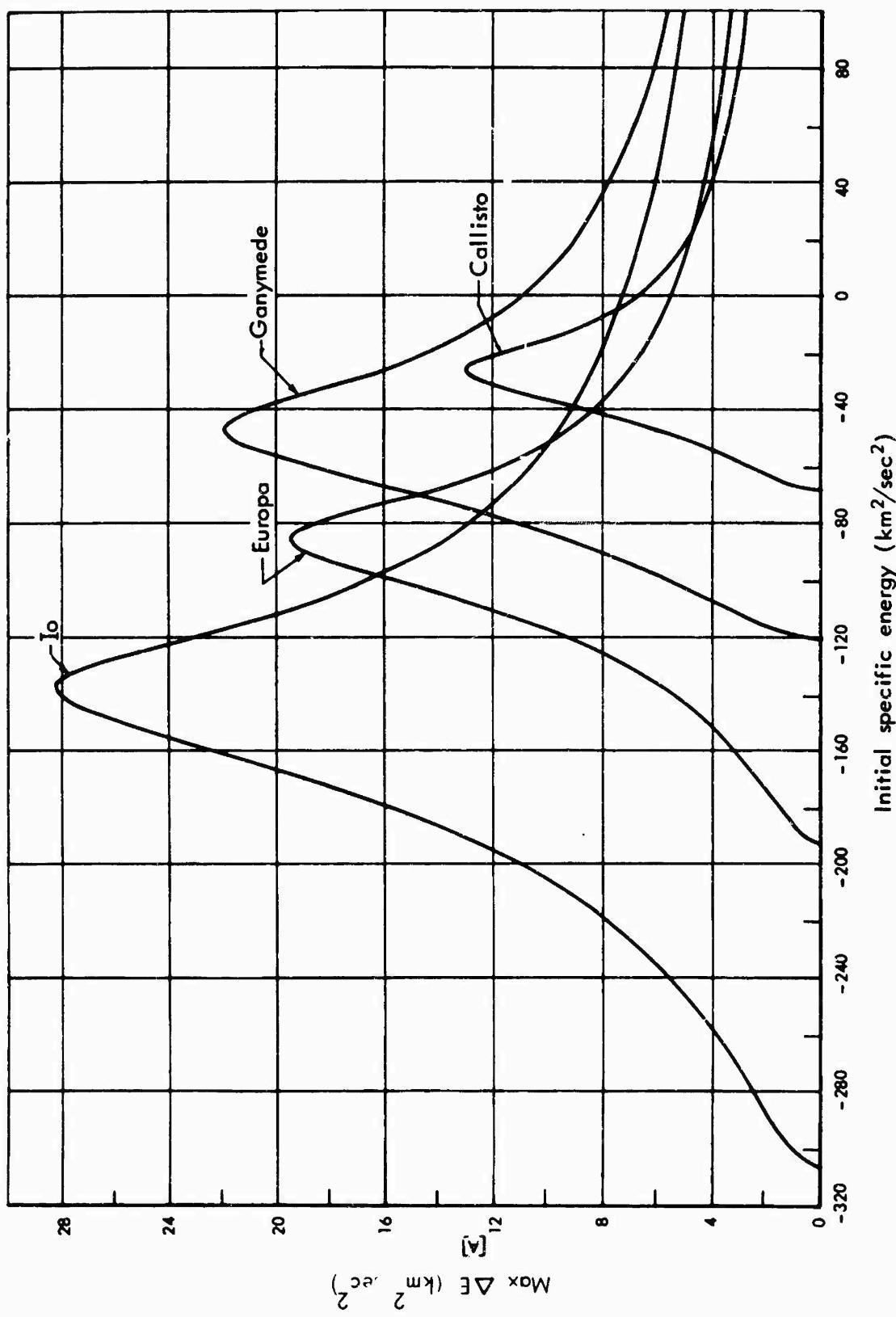


Fig. 5—Maximum possible specific energy decrease (miss ratio $M = 1$) for the four large moons as a function of the initial specific energy

a no-impulse capture resulting from a moon flyby is impossible. Therefore, either some chemical retrothrust must be applied or a low-thrust trajectory must be used. Below an incoming energy of about $9.8 \text{ km}^2/\text{sec}^2$ no-impulse captures can be accomplished by an optimal encounter, but the approach energy must be below approximately $6.0 \text{ km}^2/\text{sec}^2$ before the period of the final orbit is shorter than one year.

The changes in the magnitude of the velocity vector for initial energies of interest may vary from $\Delta|\bar{v}| = 0.72 \text{ km/sec}$ for $E = 0 \text{ km}^2/\text{sec}^2$ to $\Delta|\bar{v}| = 0.54 \text{ km/sec}$ for $E = 20 \text{ km}^2/\text{sec}^2$. The importance of such a free velocity change depends on the particular rocket used. Note that the same energy changes could be produced by chemical impulses of $\Delta|\bar{v}| = 0.18 \text{ km/sec}$ and $\Delta|\bar{v}| = 0.15 \text{ km/sec}$, respectively, if the impulses were made near the surface of Jupiter (ignoring the atmosphere and using 70,000 km as Jupiter's radius).

The above assumes that M is constrained only to be greater than unity. Since in practice this would never be attempted, and because aiming errors would cause M to deviate from any chosen nominal value, it is important to consider how much ΔE is degraded by variations in M . Figure 6 gives ΔE_{\max} versus E for Ganymede using $M \geq 1.0, 1.25, 1.5, 1.75,$ and 2.0 . As shown in Eq. (5), the height of the peaks of these curves is proportional to the inverse square root of M (and the peaks move to lower energies with increasing M , as predicted by Eq. (6)). However, in the region $E = 0$ to $E = 20 \text{ km}^2/\text{sec}^2$ the values seem to vary approximately inversely with M .

Each point in Figs. 5 and 6 results from a particular optimum approach angle A_{opt} . Figure 7 gives A_{opt} as a function of E for Ganymede. This angle--which is the magnitude of the flight path angle when the moon's orbit is circular--and ΔE determine the two possible approach trajectories that can be used to accomplish the flyby. Obviously, ΔE may also be degraded by variations in the approach angle A from the optimum angle. Figure 8 indicates the sensitivity of ΔE to A for different energies of approach. The reverse curvature observed at low energies and small approach angles occurs when $v \neq v_{\max}$. In the energy range $E = 0, 20 \text{ km}^2/\text{sec}^2$ deviations of ± 5 deg from A_{opt} cause only about 0.3 or $0.4 \text{ km}^2/\text{sec}^2$ change in ΔE . It is important to note that the

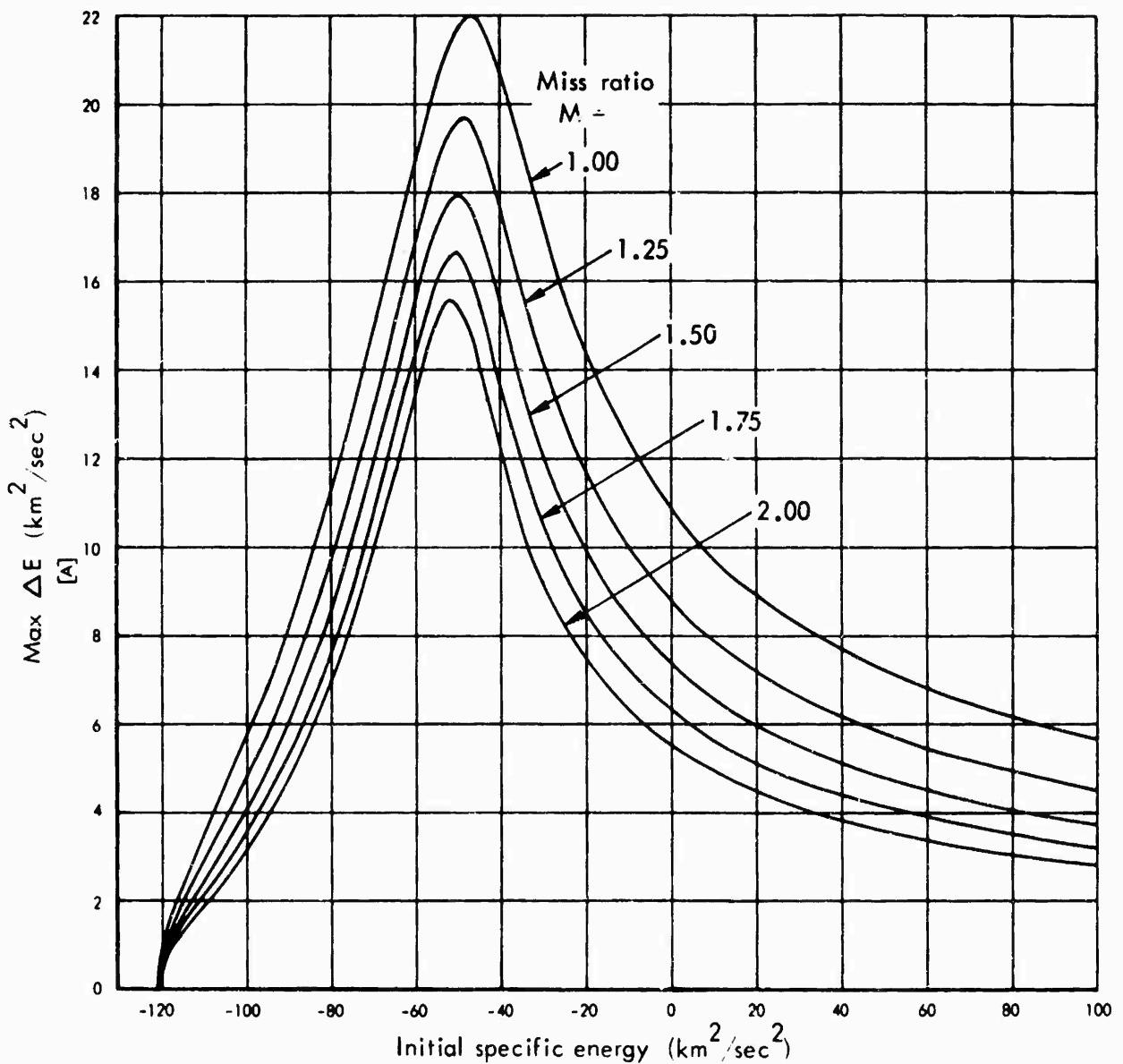


Fig. 6—Maximum possible specific energy decrease for Ganymede, with various miss ratios, as a function of the initial specific energy

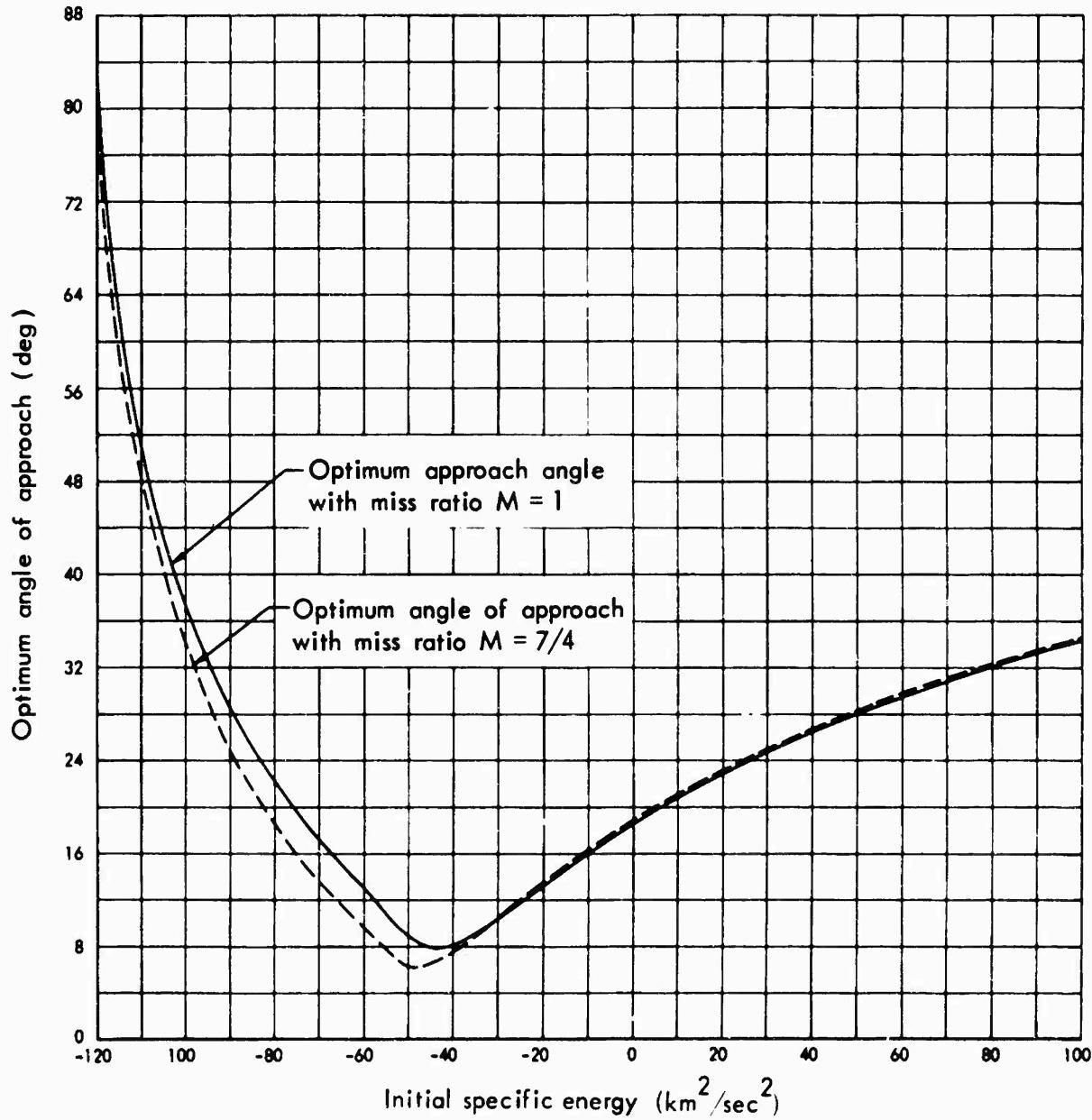


Fig. 7—Optimum angle of approach for Ganymede, with various miss ratios, as a function of the initial specific energy

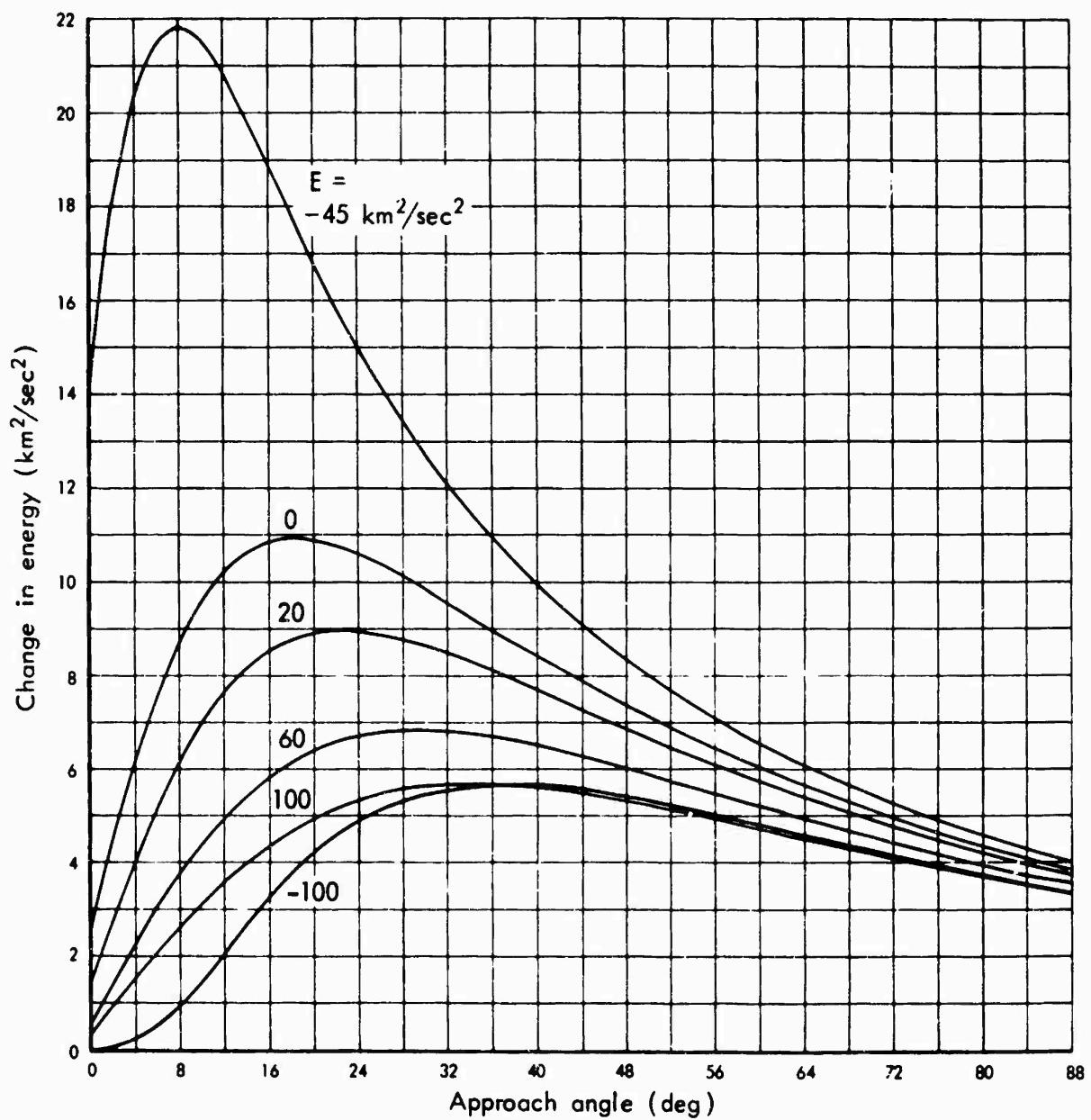


Fig. 8—Specific energy decrease for Ganymede as a function of approach angle for various initial specific energies

optimum approach angle changes only slightly with M in the energy range $E = 0, 20 \text{ km}^2/\text{sec}^2$ as shown in Fig. 7. Thus deviations in A from A_{opt} will be nearly independent of deviations in M from a nominal value.

ENERGY CHANGE RELATED TO TIMING AND AIMING ERRORS

The variation in ΔE with approach angle A can be given in a more readily understandable form by relating the approach angle to the time of arrival at the moon's radius. We define a nominal trajectory with the approach angle A_{opt} which defines a nominal approach asymptote for the approach to Jupiter. If the spacecraft arrives at the nominal point on Jupiter's sphere of influence but not at the nominal time, corrections for the change in the position of the moon must be made. We assume that the spacecraft makes these corrections perfectly. With the assumption that the sphere of influence of Jupiter is infinitely far from Jupiter, an infinitesimal velocity increment can change the approach asymptote to any desired asymptote parallel to the nominal one, as shown in Fig. 9. (Corrections made at a finite distance from Jupiter will alter the time scale in Fig. 10 and require a finite velocity increment.)

We use the fact that the semilatus rectum is equal to the square of the specific angular momentum divided by the gravitational constant for Jupiter to give the angle of approach for any energy and eccentricity:

$$A = \cos^{-1} \sqrt{\frac{a(e^2 - 1)}{r_{JM}\alpha^2}} \quad 0 \leq A \leq 180^\circ \quad (9)$$

where α is the ratio of the spacecraft's velocity at the moon's orbital radius from Jupiter to the circular velocity at that radius (the moon's velocity).

We now determine the eccentricity as a function of the time of arrival. Since the moon's orbit is circular and the direction of the perpendicular to the approach asymptote remains fixed by assumption, we can write $\psi = \psi_N + \omega_{JM}\Delta t$, where ψ_N is the nominal angle ψ , ω_{JM} is

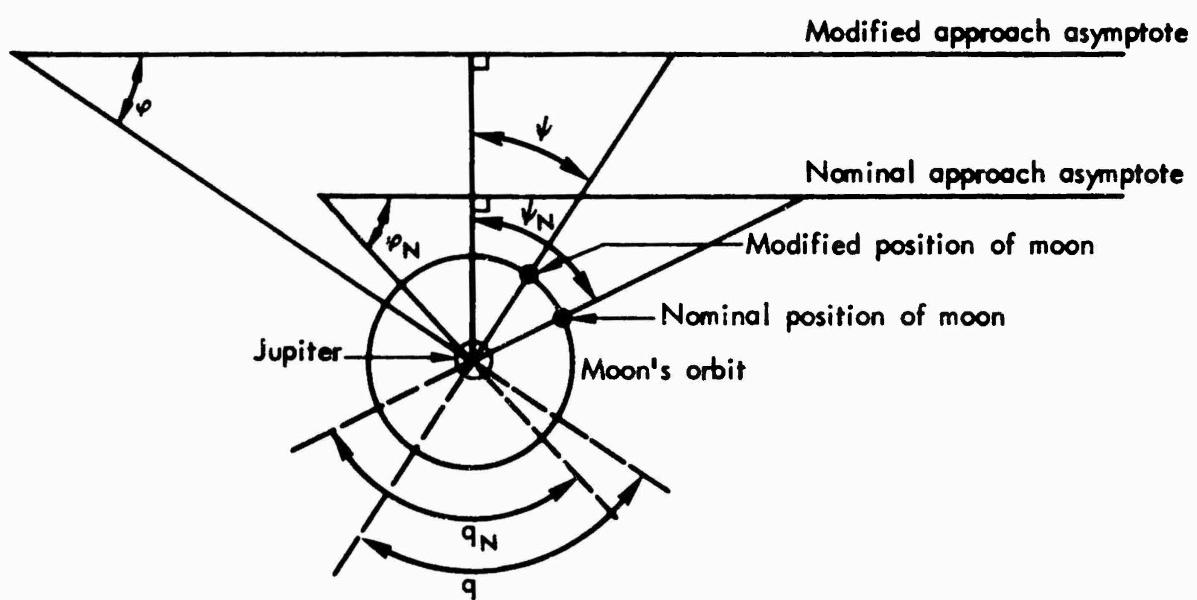


Fig. 9—Modification of approach asymptote for changes
in time of arrival

the angular velocity of the moon in its orbit, and Δt is the deviation in time of arrival at the moon's radius from the nominal time of arrival. Simple trigonometry shows that the semiminor axis of the hyperbola equals the perpendicular distance to the asymptote. Combining this with $\sin \varphi = \Delta/a\epsilon$ yields the result $\cos \varphi = 1/\epsilon$. Using this result and the fact that $q = \psi - (\varphi - 90^\circ)$ (see Fig. 9), we can write the polar equation of the orbit at the moon's orbital radius as

$$r_{JM} = \frac{a(\epsilon^2 - 1)}{1 + \epsilon \cos [\psi - \cos^{-1}\left(\frac{1}{\epsilon}\right) + 90^\circ]}$$

This is quadratic in $\sqrt{\epsilon^2 - 1}$, giving

$$\sqrt{\epsilon^2 - 1} = \frac{r_{JM} \cos \psi + \sqrt{r_{JM}^2 \cos^2 \psi + 4a r_{JM}(1 - \sin \psi)}}{2a}$$

The angle of approach as a function of time becomes

$$A = \cos^{-1} \left(\frac{r_{JM} \cos \psi(t) + \sqrt{r_{JM}^2 \cos^2 \psi(t) + 4a r_{JM}[1 - \sin \psi(t)]}}{2\alpha \sqrt{ar_{JM}}} \right) \quad (10)$$

ψ_N can be found from $\psi_N = q_N + \varphi_N - 90^\circ$ after getting q_N from the orbit equation, and using Eq. (9) to get the nominal eccentricity, which determines the nominal $\varphi = \varphi_N$.

The energy change as a function of time is determined from Eqs. (7) and (10), which were used in two example flybys of Ganymede with miss ratios of 1.0 and 1.5, and initial energies of $5 \text{ km}^2/\text{sec}^2$ relative

to Jupiter. This energy could occur after a low-thrust transfer from earth. The results of these calculations appear in Fig. 10. Note that times for which the energy change is greater than $5 \text{ km}^2/\text{sec}^2$ result in capture, although the resulting orbits may have extremely long periods. Thus for $M = 1$ we have two capture windows for arrival of the spacecraft at Jupiter that are approximately 2.4 days wide, while for $M = 1.5$ these windows are reduced to approximately 1.4 days. For times between -0.72 day and +2.4 days, the trajectories encounter the moon before reaching the pericenter of the approach orbit. For all other times pericenter is reached before the moon flyby. Of course, if it is discovered in an actual flight that the time of arrival at Ganymede's orbital radius will produce too small an energy change, it is possible that an improvement could be made by adjusting the trajectory to fly by a different moon.

The variation of ΔE with miss ratio M and angle η can also be given in a more readily understandable form. This variation establishes the degradation in ΔE due to aiming errors. Since it would be difficult to show this in general, encounters with Ganymede were considered with an initial energy of $5 \text{ km}^2/\text{sec}^2$ and a near-optimum approach angle of 19.656 deg. In Fig. 11 we look toward the moon in a direction parallel to the nominal approach asymptote, which is at the collision radius in front of the moon and in its orbital plane. We assume that any aiming errors will result in approach asymptotes that are displaced from but parallel to the nominal one. Thus any point in Fig. 11 defines an approach asymptote, and the spacecraft's orbit about the moon resulting from that asymptote will be in the plane determined by that point and the center of the moon, with its normal in the plane of the figure. The moon's orbit plane is represented by the horizontal line, and the moon's velocity has a component to the right. Thus, in general, energy decreases are obtained for asymptotes to the right of the moon, and increases are obtained for asymptotes to the left.

Figure 11 was obtained from the following considerations. It is a simple matter to relate the miss ratio to the radial distance from the moon to the asymptote Δ . Since the perpendicular distance from the moon to the asymptote is equal to the semiminor axis of the hyperbola,

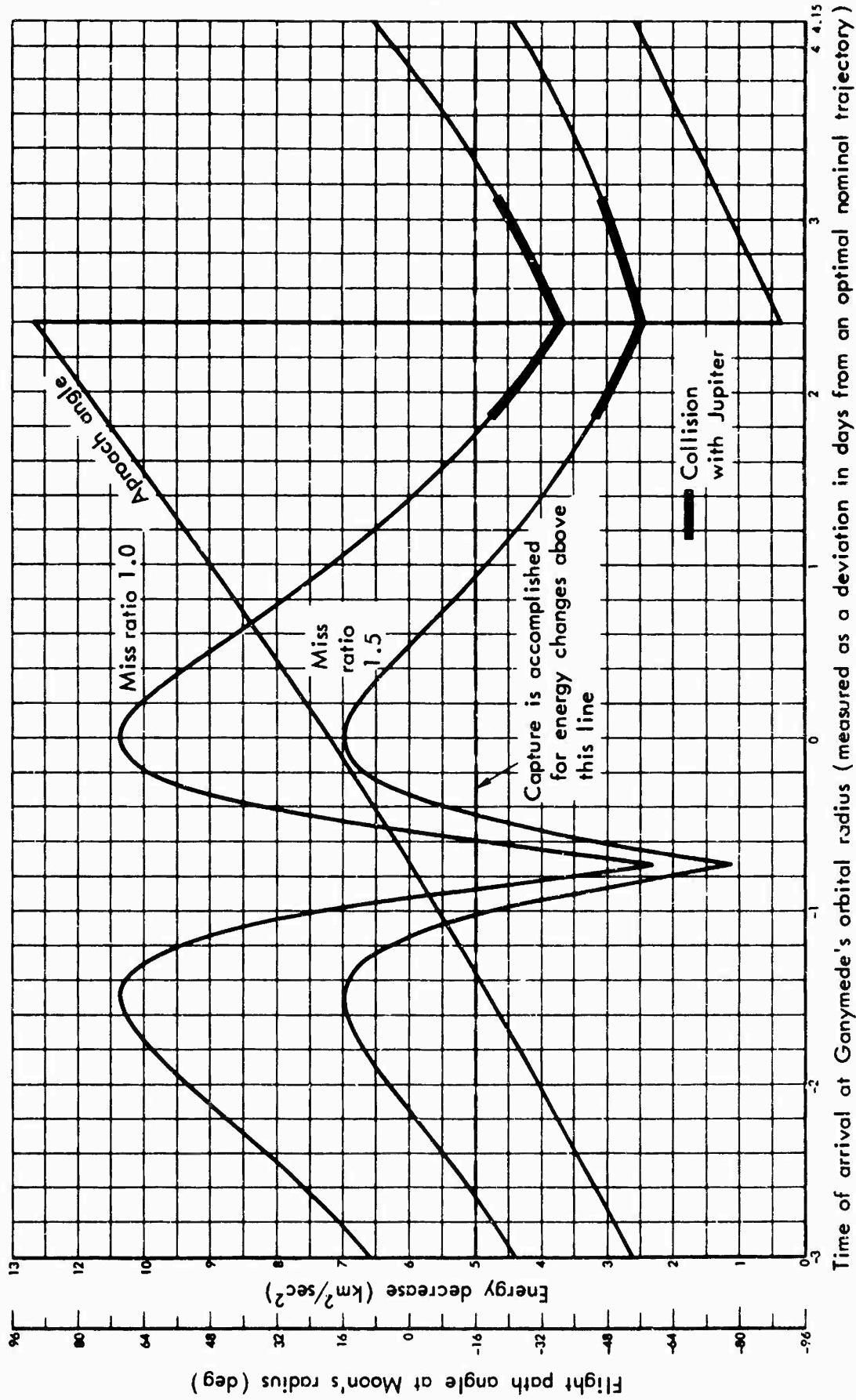


Fig. 10—Angle of approach and specific energy decrease as a function of deviations in time of arrival at Ganymede's orbital radius (measured as a deviation in days from an optimal nominal trajectory) of arrival at Ganymede's orbital radius (initial energy $5 \text{ km}^2/\text{sec}^2$)

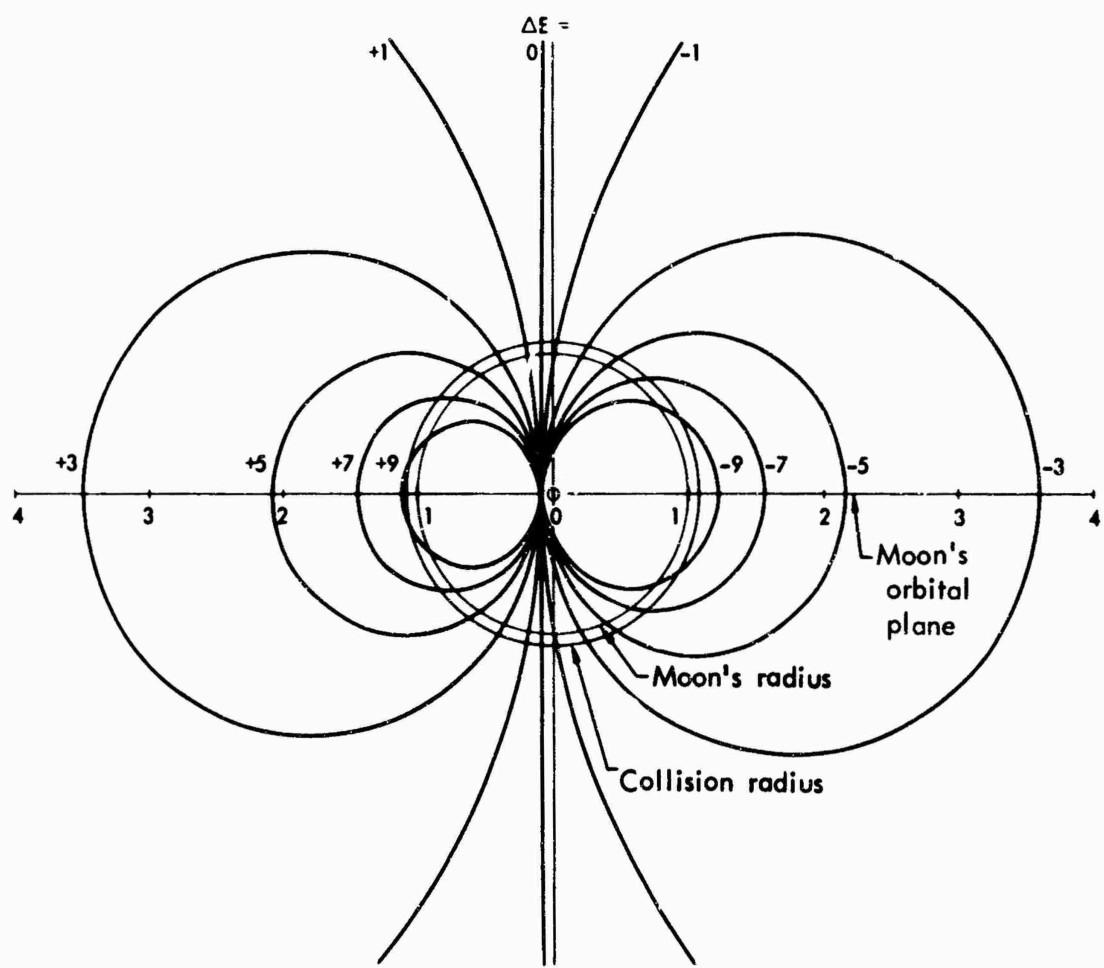


Fig. 11—Locus of approach asymptotes giving equal energy change for a Ganymede flyby. Approach angle $A = 19.565$ deg.
Initial energy, with respect to Jupiter, $5 \text{ km}^2/\text{sec}^2$.
Collision radius = 1.087 moon radii

we can relate eccentricity to Δ . We use the energy equation to relate the semimajor axis to V_∞ and combine these results to get the peri-center distance as a function of V_∞ , Δ , V_p . Thus

$$M = \frac{1}{2} \left(\frac{V_p}{V_\infty} \right)^2 \left[\sqrt{1 + 4\Delta^2 \left(\frac{V_\infty}{V_p} \right)^4} - 1 \right] \quad (11)$$

where Δ must be specified in moon radii.

Let us designate the angle between the moon's orbital plane and the plane of the satellite orbit within the sphere of influence of the moon as β . Simple trigonometry shows that

$$\sin \eta = \sin Q \sin \beta \quad (12)$$

We use this in Eq. (8) to obtain V_{JS} , V_{JM} , and A projected onto the orbital plane. We use these results and Eq. (11) in

$$\begin{aligned} \Delta E = & \frac{-2V_{JM}^*}{\left[1 + 2M \left(\frac{V_\infty}{V_p} \right)^2 \right]} \\ & \times \left\{ V_{JS}^* \cos A^* - V_{JM}^* \pm 2V_{JS}^* \sin A^* \left[M \left(\frac{V_\infty}{V_p} \right)^2 + M^2 \left(\frac{V_\infty}{V_p} \right)^4 \right]^{1/2} \right\} \quad (13) \end{aligned}$$

to obtain the energy change for any asymptote (Δ , β). (Equation (13) differs from Eq. (7) in that the latter picks the best value of M , and hence Δ , to use when M is constrained to be greater than or equal to some constant.) The sign of the last term in brackets in Eq. (13) is

taken as positive if $|\beta| < 90$ deg and negative if $|\beta| > 90$ deg. When $\beta = \pm 90$ deg this term is zero. The total energy change is found to become zero at $\beta = \pm 90$ deg only if \bar{V}_{JM} is perpendicular to \bar{V}_∞ .

Figure 11 shows the locus of all asymptotes that result in the same energy change. The moon's radius and the collision radius are also indicated. Energy changes for asymptotes within the collision radius are calculated assuming no collision occurs; they are included only to indicate the overall behavior of the loci. This graph gives a good indication of the significance of aiming errors. Obviously, vertical displacements of the asymptote about the optimum have much less effect on the energy change than horizontal displacements.

TWO-MOON ENCOUNTERS

The energy changes obtainable in a one-moon encounter are not as large as might be needed; as a result, several two-moon encounters were considered. Of course, the timing and aiming problems become much more severe for these cases. The only methods of adjusting for deviations in time of arrival (and hence in the separation angle between the moons) is to adjust the miss ratio of the first moon or introduce some rocket thrust. As shown in Table 2, the change in the flight path angle as a result of an encounter with a moon is on the order of 2 to 4 deg. Thus correcting for time-of-arrival errors by varying M has severe limitations, and of course increases in M degrade the total energy change.

Two-moon encounters can be classified into four types. The satellite can encounter the outermost moon first. Then it can encounter the second moon at either of two places, since the orbit resulting from the first encounter must intersect the second moon's orbit at two points (unless the two points happen to coincide). An encounter at either of these positions will give the same energy change, although sensitivity problems will be different. Also, the satellite can encounter the innermost moon first, at either of two points, and then the outermost moon. Again the energy change will be the same. Example trajectories for optimal encounters using an initial energy of $5 \text{ km}^2/\text{sec}^2$ are shown in Figs. 12 and 13. The changes in the orbits resulting from the swingbys

Table 2
ORBIT CHANGES FROM TWO-MOON ENCOUNTERS

Item	E (km 2 /sec 2)	A (deg)	V_{JS} (km/sec)	ϵ	q (deg)	τ^{δ} (earth solar days)	a (Jupiter radii)	r pericenter (Jupiter radii)
Ganymede-Callisto								
Initial orbit approaching Ganymede	5.00	18.25	15.86	1.075	35.19	∞	-181.2	13.58
After Ganymede encounter	-5.32	21.65	15.20	0.924	45.18	265.64	170.2	12.87
Values approaching Callisto	--	44.50	11.25	--	--	--	--	--
After Callisto encounter	-10.33	46.60	10.80	0.871	103.11	98.24	87.7	11.32
Callisto-Ganymede								
Initial orbit approaching Callisto	5.0	42.0	12.14	1.083	80.16	∞	-181.2	15.02
After Callisto encounter	0.14	44.03	11.73	1.002	87.96	∞	-6645.0	13.66
Values approaching Ganymede	--	17.42	15.55	--	--	--	--	--
After Ganymede encounter	-10.76	21.02	14.83	0.847	46.08	92.35	84.2	12.89

δ Orbital period.

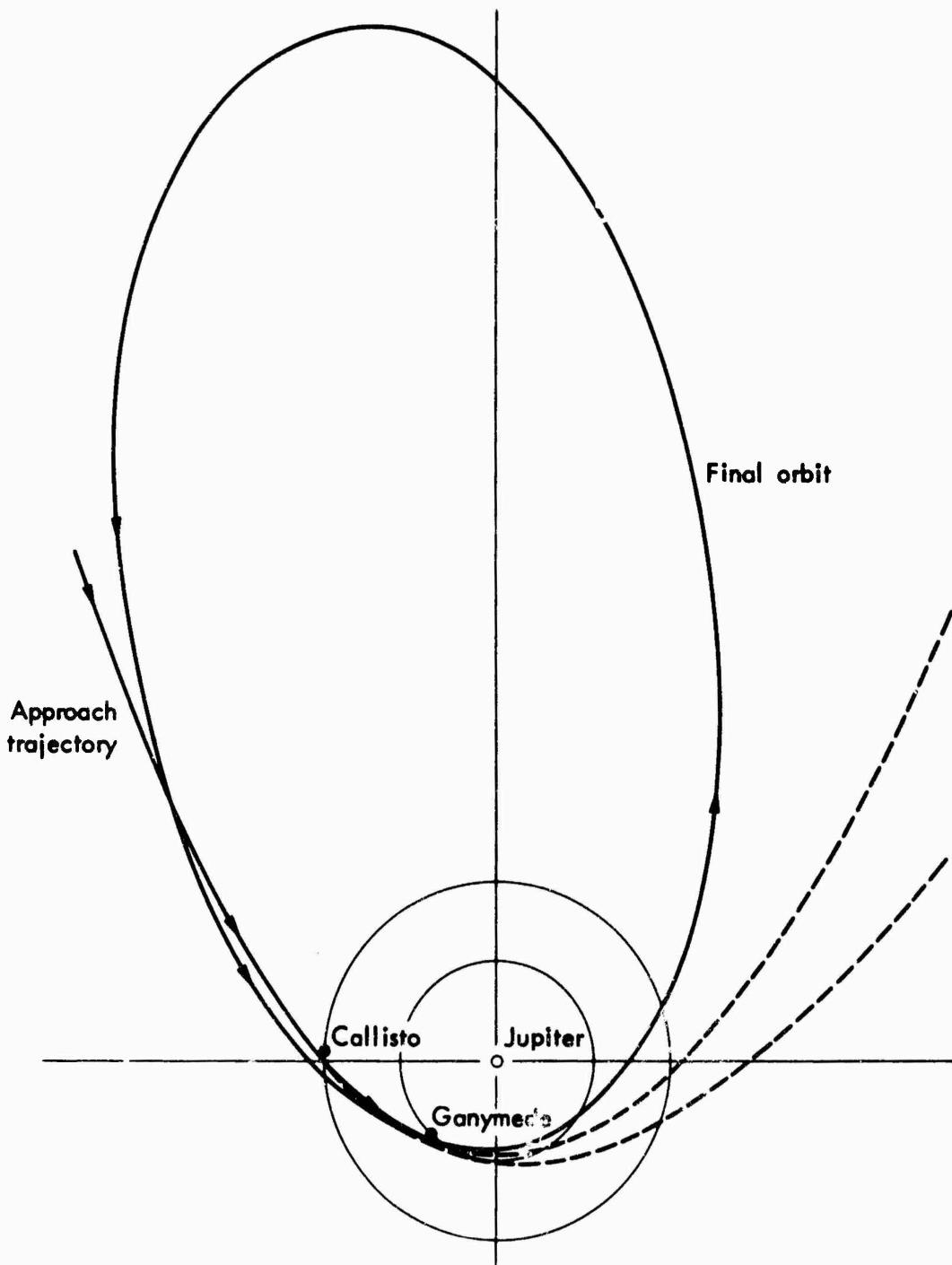


Fig. 12—Example of two-moon encounters for initial specific energy
5 km²/sec², Callisto-Ganymede

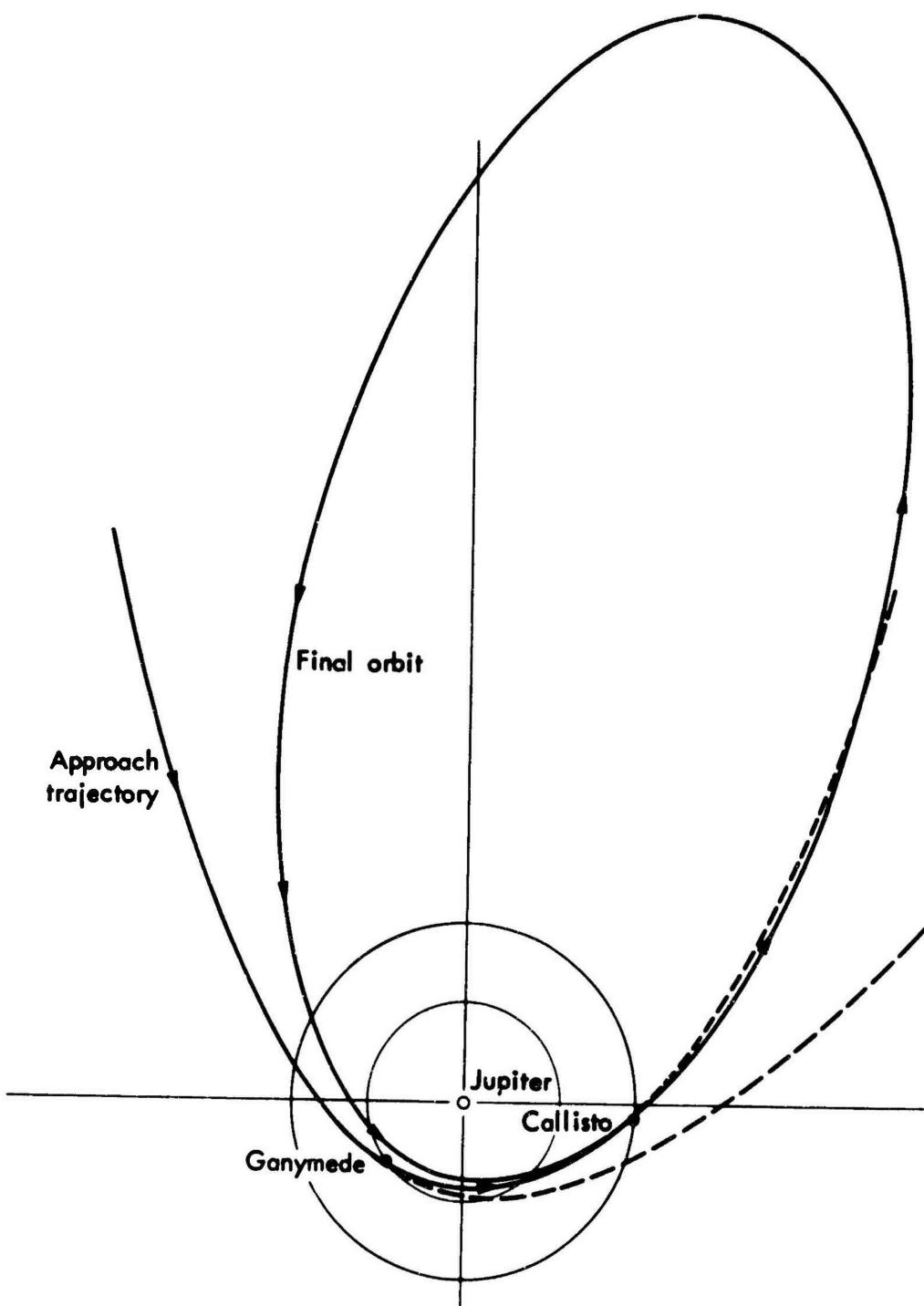


Fig. 13—Example of two-moon encounters for initial specific energy
 $5 \text{ km}^2/\text{sec}^2$, Ganymede-Callisto

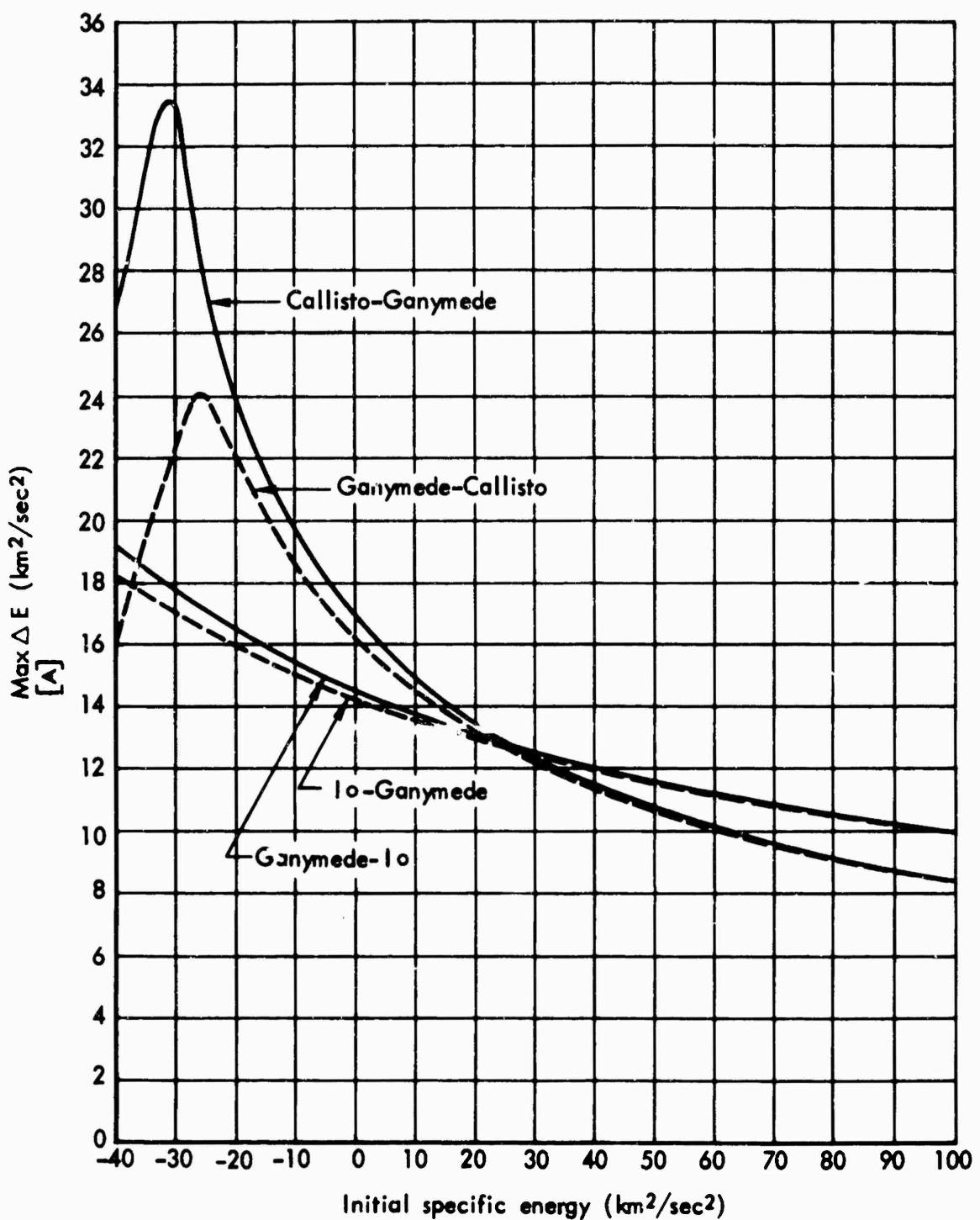


Fig. 14—Maximum possible specific energy decrease for various two-moon encounters assuming the moons are oriented as needed

are shown in Table 2. The maximum possible energy changes ($M \geq 1$ for each moon) for different approach energies are given in Fig. 14 and the corresponding optimum angles of approach in Fig. 15. For each initial energy and approach angle, a given angular relationship between

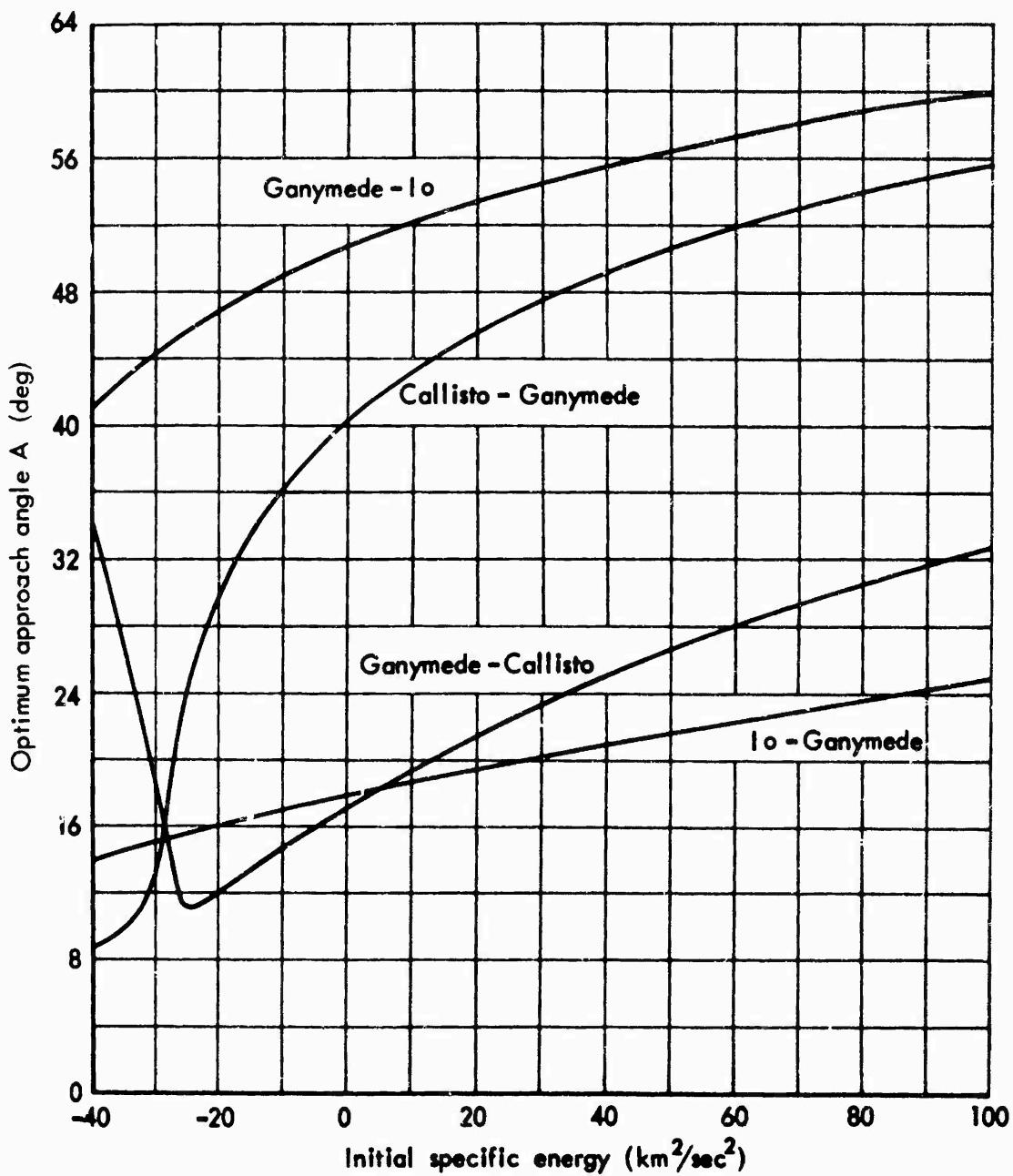


Fig. 15—The optimum approach angle for two-moon encounters

the two moons is implied. Because the periods of revolution of the moons are rather short, the required angular relationship may be found fairly frequently. It is more difficult to find an opportunity to effect the required approach angle to the first moon when the angular relationship between the moons is correct.

IV. CONCLUSIONS

The energy change obtainable from a moon encounter constitutes at most less than half the necessary energy change for high-thrust approach trajectories according to the simplified model used. This energy change may not be sufficiently large to justify the use of a moon swingby (unless, of course, one of the mission objectives is observation of a moon). However, for the low-thrust case these flybys are considerably more interesting. The guidance problems associated with a swingby are probably more easily handled by low-thrust vehicles, and since for these vehicles it is more difficult to create significant velocity changes close to the surface of Jupiter, the free velocity change from a moon, in terms of resulting energy change, becomes correspondingly more important. Furthermore, the relaxation of the required velocity endpoint conditions for low-thrust trajectories may allow significantly increased payloads or shorter flight times.

The energy change is found to vary significantly with variations in the time of arrival. Arriving three-fourths of a day ahead of the nominal time in Fig. 10 cuts the energy change to less than one-fourth its nominal value. The arrival windows for the case considered are 2.4 days for a miss ratio $M = 1$ and 1.4 days for a miss ratio of 1.5. Encounters with Io or Europa will have shorter arrival windows for the same approach energy due to their shorter orbital periods and smaller energy changes.

Trajectories which use gravity assist from two moons give much larger energy changes but also present much more difficult timing and aiming problems.

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